GEOPHYSICS, VOL. 62, NO. 1 (JANUARY-FEBRUARY 1997); P. 319–324, 2 FIGS., 1 TABLE.

Short Note

Seismic attenuation in finely layered porous rocks: Effects of fluid flow and scattering

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INTRODUCTION

Scattering caused by fine layering is known to play an important role in seismic attenuation. If the rock is also porous and permeable, additional attenuation can be caused by a flow of the pore fluid induced by the passing wave (see e.g., White, 1983). At frequencies higher than 100 kHz, standard Biot's attenuation (Biot, 1962; Bourbié et al., 1987) may also contribute to the overall attenuation in a layered poroelastic rock. While standard Biot's attenuation has been well known for decades, the effects of scattering by fine layering and interlayer flow have been studied extensively both theoretically and numerically in recent years. However, in all theoretical approaches proposed thus far, these effects have been treated separately: scattering was studied for purely elastic rocks (Burridge and Chang, 1989; Shapiro et al., 1994), while attenuation caused by interlayer flow was considered either for periodically stratified media that causes no scattering (Norris, 1993) or for frequencies at which scattering is negligible (Gurevich and Lopatnikov, 1995).

In this short note, we perform the theoretical and numerical study of the seismic attenuation in finely layered poroelastic rocks, caused by the combined effect of interlayer flow and scattering, as well as standard Biot's visco-inertial mechanism (for simplicity, only waves propagating normal to layering are considered). The known theoretical solutions for these three mechanisms of attenuation are analyzed in comparison for randomly and periodically layered poroelastic media. A quantitative example is used to identify the relative magnitude, predominant frequency range, and typical frequency dependence for each of these mechanisms. For situations where the contributions of different mechanisms are comparable, we propose that their combined effect can be adequately modeled by superposing the theoretical solutions for the three phenomena.

To test the approximate theoretical solutions, we perform numerical experiments using the OASES-Biot software package that simulates elastic wave propagation in a sequence of solid, fluid, and poroelastic horizontal layers. For all the three mechanisms of attenuation, the numerical results are in good agreement with the theoretical predictions. The numerical tests also confirm that the superposition model gives a satisfactory approximation of the overall attenuation.

THEORY

Biot equations for 1-D medium

Elastic wave propagation through a finely layered poroelastic medium may be described by Biot equations of poroelasticity. Consider a porous continuum consisting of a material of solid grains with the bulk modulus K_s and density ρ_s, and a pore fluid with the bulk modulus (incompressibility) K_f, dynamic viscosity η, and density ρ_f. The solid grains form an elastic matrix that is characterized by the porosity ϕ, permeability κ, and bulk and shear moduli K and μ, measured in dry (unsaturated) conditions. The parameters of the medium are assumed to depend on the z coordinate only. For time-harmonic compressional waves propagating along z-axis the Biot equations of poroelasticity can be written in the form (Biot, 1962)

$$\frac{d}{dz}\left(H\frac{du}{dz}+C\frac{dw}{dz}\right)+\omega^2(\rho u+\rho_f w)=0, \qquad (1)$$

and

$$\frac{d}{dz}\left(C\frac{du}{dz}+M\frac{dw}{dz}\right)+\omega^2(\rho_f u+qw)=0. \qquad (2)$$

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Here u is the solid displacement, and $w = \phi(u_f - u)$ is the weighted (by porosity) fluid displacement relative to the solid matrix, with u_f denoting the fluid displacement (the scalar form is used since only z-components of the displacements are considered). The overall density of the two-phase medium is defined as $\rho = \phi\rho_f + (1-\phi)\rho_s$, and the coefficients q, H, C, and M are given by

$$q = i\eta F/\kappa\omega, \tag{3}$$

$$H = K + \tfrac{4}{3}\mu + \sigma C, \tag{4}$$

$$C = \sigma M, \tag{5}$$

$$M = 1\bigg/\left[\frac{\sigma-\phi}{K_s} + \frac{\phi}{K_f}\right], \tag{6}$$

and

$$\sigma = 1 - K/K_s. \tag{7}$$

Here and below the time dependence $e^{-i\omega t}$ is implicit ($\omega = 2\pi f$).

In equation (3), $F = F(\omega/\omega_c)$ is the dynamic correction function, and $\omega_c = \eta\phi/\kappa\rho_f$ is a so-called Biot's characteristic frequency. For a homogeneous medium the characteristic equation of the linear system (1)–(2) is a familiar dispersion equation for the compressional waves in a *homogeneous* fluid-saturated porous medium (Biot, 1962; Bourbié et al., 1987). The roots of this equation are the (complex) wavenumbers of the fast and slow (type II) compressional waves $\tilde{k}_1$ and $\tilde{k}_2$. At low frequencies that obey the condition $\omega \ll \omega_c$, the function $F \approx 1$, and the wavenumbers $\tilde{k}_1$ and $\tilde{k}_2$ may be expressed in the form

$$\tilde{k}_1 = k_1 + i\alpha_B = k_1\left(1 + iQ_B^{-1}/2\right), \tag{8}$$

$$\tilde{k}_2 = \omega\left(\frac{q}{N}\right)^{1/2} = \left(\frac{i\omega\eta}{\kappa N}\right)^{1/2} = (1+i)k_2. \tag{9}$$

Here

$$k_1 = \mathrm{Re}\{\tilde{k}_1\} = \omega/c_1 = \omega\left(\frac{\rho}{H}\right)^{1/2}, \tag{10}$$

$$k_2 = \mathrm{Re}\{\tilde{k}_2\} = \left(\frac{\omega\eta}{2\kappa N}\right)^{1/2}; \tag{11}$$

$c_1 = (H/\rho)^{1/2}$ is the phase velocity of the normal (fast) P-wave, $\alpha_B = \omega Q_B^{-1}/2c_1$ is its amplitude attenuation coefficient, and the constant N is defined by

$$N = (MH - C^2)/H = \frac{M}{H}\left(K + \frac{4}{3}\mu\right). \tag{12}$$

For well-consolidated rocks the dimensionless attenuation of the fast wave Q_B^{-1} can be approximately written as

$$Q_B^{-1} = \frac{\rho_f^2}{\rho|q|}\left(1 + \frac{\rho^2 M}{\rho_f^2 H} - 2\frac{\rho C}{\rho_f H}\right) \approx \frac{\rho_f}{\rho}\frac{\omega}{\omega_c}\phi. \tag{13}$$

This Biot's attenuation is caused by the viscous dissipation in the oscillatory flow of the pore fluid relative to the solid matrix, the flow that is induced by the passing wave because of the difference in properties between the solid and the fluid.

It turns out from Biot theory that at low frequencies the compressional type II wave is indeed slow compared with the normal compressional (type I or "fast") wave which, in fact, is a normal P-wave with very small attenuation, so that $k_1 \ll k_2$ and $Q_B^{-1} \ll 1$.

Random and periodic layering

According to Biot theory, at low frequencies the attenuation of the normal P-wave in a homogeneous poroelastic medium is caused by the so-called global flow phenomenon. The passing wave induces small fluid pressure gradients between regions of compression and extension, and these gradients cause fluid flow relative to the solid. The fluid flow incurs viscous loss, resulting in a small attenuation of the passing wave.

In real rocks such phenomena may be overridden by so-called local flow effects, caused by the flow of the pore fluid between regions of different compliances under the compression (or extension) induced by the passing wave (Mavko and Nur, 1975; Jones, 1986). If the medium is composed of thin alternating layers of two poroelastic materials with different compliances, then propagation of a P-wave will squeeze the fluid from the more compliant into the less compliant layers. This local flow of the pore fluid is accompanied by the viscous loss and results in the attenuation of the passing wave.

The local flow attenuation is not taken into account by the traditional approach to Biot theory, the approach that assumes the spatial homogeneity of a poroelastic medium. However, if spatial homogeneity is not assumed, Biot theory can be used effectively to analyze wave propagation in inhomogeneous poroelastic media. Such a theory will implicitly account for the local flow attenuation.

Equations (1) and (2) govern the propagation of compressional waves in a poroelastic medium with an arbitrary dependence of its properties on the coordinate z. The effect of fine layering, either random or periodic, on the attenuation can be studied in the context of these equations by assuming that properties of the medium [or the coefficients of equations (1) and (2)] are random or periodic functions of z. Although in physics random and periodic situations are usually treated quite differently, it is also possible to treat a periodic function as a random function with a periodic autocorrelation. For the sake of uniformity, the latter approach is used in this paper. As an aside, we will show that the results so obtained agree well with the exact solutions known for periodically layered media. Based on the above, we assume that any given parameter of the medium, say, ζ, is a random function of z with a constant average $\langle\zeta\rangle$, and the normalized autocorrelation function $\psi(\xi) = \langle\epsilon(z)\epsilon(z+\xi)\rangle$, where $\epsilon(z)$ denotes relative fluctuations of the parameter ζ, i.e., $\zeta = \langle\zeta\rangle[1 + \epsilon(z)]$. Here and below angle brackets denote statistical (ensemble) averaging. The type of layering is defined by the form of the normalized autocorrelation function. For random layering we will assume the exponential correlation

$$\psi(\xi) = \langle\epsilon^2\rangle \exp(-2\xi/h). \tag{14}$$

For periodic layering, the following "serrate" correlation function has proved appropriate:

$$\psi(\xi) = \langle\epsilon^2\rangle \times \begin{cases} 1 - 2(\xi - 2nh)/h, & 2nh \le \xi < (2n+1)h \\ -1 + 2[\xi - (2n+1)h]/h, & (2n+1)h \le \xi < 2(n+1)h. \end{cases} \tag{15}$$

Once such an assumption about the statistics of layering has been made, the problem of wave propagation through a finely layered poroelastic medium reduces to a stochastic system of equations which could, in principle, be analyzed by rigorous stochastic equation methods. However, for such complex equations as Biot equations of poroelasticity, this approach appears to be too complex, given the complexity of the analogous problem for elastic media (Shapiro and Hubral, 1994). To avoid such a complexity, we propose a certain decomposition of the problem on the basis of known physics of the phenomenon.

As pointed out in several publications, attenuation of a compressional wave propagating through a finely layered porous rock can be caused by the three factors: (1) classical viscous-inertial Biot attenuation; (2) scattering owing to fine layering; (3) wave-induced flow of the pore fluid from layer to layer. Our idea of the decomposition is to treat the phenomena (1)–(3) separately, and to use the superposition of the results as a solution of the overall problem. In the following, we will outline the theoretical solutions for the three mechanisms of attenuation to be used in the superposition model.

Standard Biot's attenuation.—As mentioned above, Biot theory predicts a small attenuation of a normal (fast) P-wave in a homogeneous poroelastic medium. Obviously, this attenuation will also take place in a layered poroelastic medium, but will not be affected significantly by the layering.

Scattering attenuation.—Scattering attenuation in our model is caused by fine layering, and is not supposed to be affected by poroelasticity. Thus, to focus on the effect of scattering in a poroelastic layered medium, we can get rid of the poroelastic effects if we replace each individual poroelastic layer with an equivalent elastic layer with the density $\rho(z)$ and elastic velocities $v_p(z) = c_1(z) = \sqrt{H(z)/\rho(z)}$, and $v_s(z) = \sqrt{\mu(z)/\rho(z)}$. By doing so, we reduce the problem of attenuation in a finely layered poroelastic medium to a problem of attenuation in an equivalent finely layered elastic medium. Moreover, we consider only waves propagating normally to layering, and therefore our problem is also equivalent to that for an acoustic finely layered medium.

Consider P-waves propagating through a layered medium with a given correlation function $\psi(\xi)$ of the fluctuations $\epsilon(z)$ of the P-impedance,

$$v_p(z)\rho(z) = \langle v_p(z)\rho(z)\rangle[1 + \epsilon(z)]. \tag{16}$$

The solution for the scattering attenuation in such a medium is known as a generalized O'Doherty–Anstey formula (Shapiro et al., 1994), which for waves propagating normal to layering can be written in the form

$$Q_{\text{scat}}^{-1} = 2k\int_0^\infty d\xi\, \psi(\xi)\cos(2k\xi), \tag{17}$$

where $k = \omega/\langle v_p\rangle = k_1$.

As can be seen from equation (17), the scattering attenuation depends essentially on the form of the function $\psi(\xi)$. For random fluctuations with exponential correlation (14), the result is given in Shapiro et al. (1994) as

$$Q_{\text{scat}}^{-1} = S\frac{kh}{1 + k^2h^2}, \tag{18}$$

where $S = \psi(0) = \langle\epsilon^2\rangle$.

The solution for the scattering attenuation in a randomly layered medium as a function of frequency given by equation (18) is a typical relaxation curve (long-dashed line in Figures 1 and 2) with the maximum occurring at a frequency

$$f_{\text{scat}} = c_1/2\pi h, \tag{19}$$

at which $k_1h = 1$. In the low-frequency limit ($f \ll f_{\text{scat}}$) the (dimensionless) scattering attenuation Q_{scat}^{-1} is proportional to f^1, while at higher frequencies ($f \gg f_{\text{scat}}$) it decreases with increasing frequency as f^{-1}.

Equation (17) can be used, at least technically, to derive an expression for the attenuation in a periodically layered medium. To do so, we have to substitute the correlation

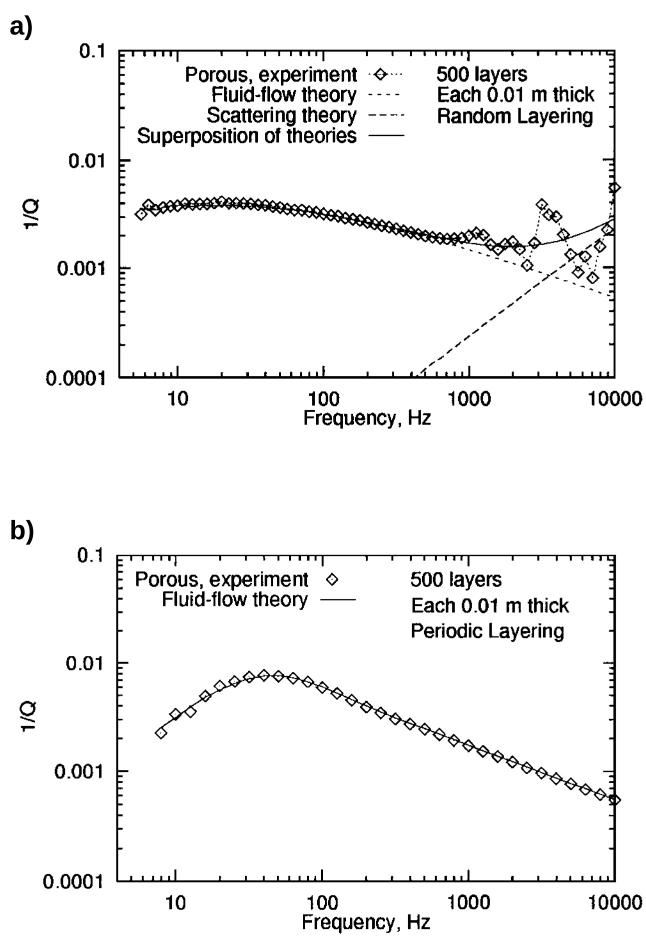

FIG. 1. Attenuation (inverse Q) for an extremely finely layered water-saturated sandstone, composed of alternating layers of two low-permeability (0.5–0.8 $\times 10^{-14}$ m^2) poroelastic materials, whose rock matrix properties are shown in Table 1 under (1) and (2). (a) random layering, (b) periodic layering.

function (15) into equation (17). The integration is most easily performed if we express the function (15) in Fourier series,

$$\psi(\xi) = \langle\epsilon^2\rangle \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{\cos[(2n+1)\pi x/h]}{(2n+1)^2}. \qquad (20)$$

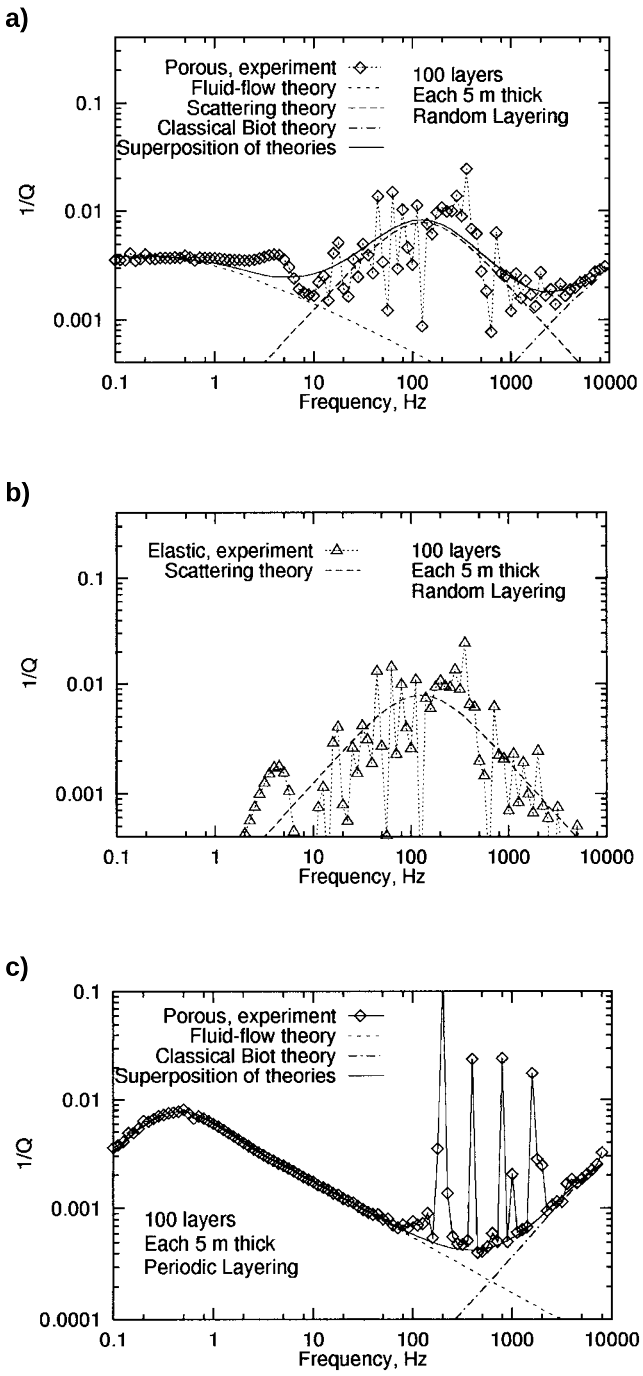

FIG. 2. Attenuation (inverse Q) for a finely layered water-saturated sandstone, composed of alternating layers of two high-permeability (0.5–0.8 $\times 10^{-12}$ m^2) poroelastic materials, whose rock matrix properties are shown in Table 1 under (1) and (2). (a) and (b) random layering; (c) periodic layering.

Thereby, the integration reduces to a Fourier transformation of cosine functions, and yields

$$Q_{\text{scat}}^{-1} = \frac{4Sk}{\pi} \sum_{n=-\infty}^{\infty} \frac{1}{(2n+1)^2} \delta\left[k - \frac{(2n+1)\pi}{2h}\right]. \quad (21)$$

Equation (21) shows that the attenuation in a periodically stratified elastic medium is zero, except for the odd multiples of the resonant frequency $f_0 = c/4h$, where no propagation can take place. This result is consistent with the exact solution of the problem of wave propagation in a periodic medium (Brillouin, 1963), considered in the limit of low contrast of medium properties between layers. The "band-stop" frequencies are those at which constructive interference of backwards reflected waves occurs. The solution (21) implies that there is no scattering of energy in periodic media: all the energy is either fully transmitted or fully reflected backwards.

Attenuation caused by the fluid flow between layers.—The solution for the attenuation caused by the pore fluid flow between layers of a finely layered poroelastic medium (without scattering) can be obtained by neglecting all scattering terms in stochastic equations derived from equations (1) and (2). The simplified stochastic equations can be then analyzed by using a simple statistical technique known as Bourret approximation. This has been done by Gurevich and Lopatnikov (1995) who obtained the following solution for the attenuation of the normal P-wave caused by the pore fluid flow between layers in a poroelastic finely layered medium:

$$Q_{\text{flow}}^{-1} = \sqrt{2}sk_2 \int_0^{\infty} d\xi\ \psi(\xi) \exp(-k_2\xi) \cos(k_2\xi + \pi/4), \quad (22)$$

where the constant s is defined by

$$s = \frac{\langle C/H\rangle}{\langle 1/H\rangle\langle N\rangle}. \qquad (23)$$

Here $\psi(\xi)$ is the autocorrelation function of ϵ which now denotes the fluctuations of C/H ratio,

$$C(z)/H(z) = \langle C/H\rangle[1 + \epsilon(z)]. \qquad (24)$$

Similar to the scattering case, explicit expressions for random and periodic layering can be obtained by the substitution of an appropriate correlation function into equation (22). For the medium with the exponential correlation we get

$$Q_{\text{flow}}^{-1} = s^* \frac{k_2 h}{(k_2 h)^2 + 2k_2 h + 2}, \qquad (25)$$

where $s^* = s\langle\epsilon^2\rangle$. Similarly, the substitution of the periodic correlation function (15) into equation (22) yields

$$Q_{\text{flow}}^{-1} = s^* \frac{\sinh k_2 h - \sin k_2 h}{k_2 h(\cosh k_2 h + \cos k_2 h)}. \qquad (26)$$

As shown in Gurevich and Lopatnikov (1995) this result is in good agreement with the exact solution for periodically stratified poroelastic layers, which was recently published in Norris (1993).

Using equations (25)–(26) along with equation (11), we can analyze the attenuation caused by fluid flow as a function of frequency (short-dashed line in Figures 1 and 2). Both for random and periodic layering the function $Q_{\text{flow}}^{-1}(f)$ has its maximum in the vicinity of a frequency

$$f_{\text{flow}} = \frac{\kappa}{\eta} \frac{K_f}{\phi h^2}, \tag{27}$$

at which $k_2 h \approx 1$. This result had to be expected since the interlayer flow is associated with the energy conversion from the normal (fast) P-wave to the slow P-wave at the interfaces, as shown by Gurevich and Lopatnikov (1995). In the low-frequency limit, Q_{flow}^{-1} is proportional to $\sqrt{f}$ for random layering with exponential correlation and to f for periodic layering. At frequencies higher than f_{flow} attenuation decreases with increasing frequency as $1/\sqrt{f}$ for both kinds of layering.

Superposition of the theoretical results

The lines in Figures 1 and 2 schematically show the behavior of the inverse Q versus frequency for the three mechanisms of P-wave attenuation in a randomly layered poroelastic medium. The maximum scattering attenuation occurs when the characteristic layer thickness h (twice the correlation length) equals $\lambda_1/2\pi$, where $\lambda_1 = 2\pi/k_1$ is the wavelength of the propagating wave (ordinary P-wave), whereas for the fluid flow attenuation the maximum occurs when the same quantity h roughly equals the attenuation length $\lambda_2 = 1/k_2$ of the type II or "slow" wave. Since $k_2 \gg k_1$, the characteristic frequency for fluid flow attenuation f_{scat} is usually substantially higher than f_{flow}. The standard Biot attenuation builds up at still higher frequencies. Nevertheless, in many cases the curves representing the attenuation owing to the three mechanisms overlap (as shown in Figure 2a). In such circumstances, it seems reasonable to assume that the attenuation caused by the three mechanisms would make up the superposition of the three solutions

$$Q^{-1} = Q_B^{-1} + Q_{\text{scat}}^{-1} + Q_{\text{flow}}^{-1}. \tag{28}$$

This superposition model is shown as a solid line in Figures 1a and 2a.

For a periodically layered medium, the theoretical solutions for the three mechanisms of attenuation are shown in Figures 1b and 2c. However, the solution for the scattering effect is not shown. As mentioned above, there is no scattering in a periodic medium. Thus, the solution (21) cannot be included in the superposition model for periodic media, which in this case reads

$$Q^{-1} = Q_B^{-1} + Q_{\text{flow}}^{-1}. \tag{29}$$

This is the model shown as a solid line in Figures 1b and 2c.

NUMERICAL TESTS

Modeling

The theoretical solutions for the scattering and fluid-flow attenuation presented in the previous section are essentially statistical in nature, and are based on a number of assumptions (infinite extent of the layered medium; small contrast between layers; additivity of the attenuation mechanisms). To check the validity of the theoretical solutions, we perform a set of numerical experiments for a finite stack of poroelastic layers. The modeling is performed using the software package OASES (Schmidt and Tango, 1986) which allows computation of the time-harmonic plane-wave transmission coefficients for elastic waves propagating through a sequence of horizontal layers. Recently developed so-called Biot extension of OASES enables handling of porous fluid-saturated layers, in addition to elastic and fluid layers, treated as Biot-type two-phase continua with open-pore boundary conditions at interfaces (Deresiewicz and Skalak, 1963).

We use this modeling code to compute transmission coefficients T of waves propagating through a computer-generated random (or periodic) sequences of fluid-saturated layers. Corresponding attenuation coefficients α and inverse quality factors $1/Q$ are then computed using simple formulas $T = \exp(-\alpha h_\Sigma)$, $1/Q = \alpha c_1/\pi f$, where h_Σ is an overall thickness of the stack of layers, and c_1 is a Backus average velocity in the stratified medium. The same procedure is also performed for an equivalent randomly layered elastic sequence (porous layers are replaced by equivalent elastic ones using the Gassmann equation).

Results

Numerical experiments are performed for random and periodic stacks of n alternating layers of porous materials (1) and (2) (Table 1). The random stack is generated by throwing $n-1$ points z_j randomly (using RND with the uniform distribution) onto the interval $0 < z < h_\Sigma$, and rearranging them in ascending order. Each element z_j of the sequence so obtained $0 < z_0 < z_1 < \cdots < z_{n-1} < h_\Sigma$ is then taken as a location on the z-axis of jth interface between the materials (1) and (2), or vice versa. The resulting function $\epsilon(z)$ is a piecewise constant function taking the values ϵ_0 and $-\epsilon_0$ with $n-1$ transitions between these values at random points. Such a function $\epsilon(z)$ may be treated as a realization of a two-state random process (so-called telegraph process), which for $n \gg 1$ has an exponential autocorrelation function $\psi(\xi) = \epsilon_0^2 \exp(-2\xi/h)$ with the variance ϵ_0^2 and the correlation length equal to half the average layer thickness h (Korn and Korn, 1968, 18.11-5). For the periodic stack, the sequence z_j is taken equidistant: $z_j = jh$, where $h = h_\Sigma/n$ is a thickness of each individual layer. Thus, in the periodic case $\epsilon(z)$ is a *periodic* (nonrandom) piecewise constant function with values ϵ_0 and $-\epsilon_0$ and a period $2h$.

In the first example (Figure 1), we have deliberately lowered the permeability of both rocks 1 and 2 as given in Table 1 by two orders of magnitude. At the same time, we have chosen the individual layer thickness to be 1 cm. As can be seen from equations (19) and (27), this artificial choice of parameters puts the two peak frequencies f_{flow} and f_{scat} far away from

Table 1. Mechanical properties of the sample rocks 1 and 2.

Parameter	Units	(1)	(2)
Bulk modulus of dry matrix K	10^9 Pa	20.0	10.0
Shear modulus of dry matrix μ	10^9 Pa	20.0	10.0
Bulk modulus of solid grains K_s	10^9 Pa	35.0	35.0
Density of solid grains ρ_s	10^3 kg/m^3	2.65	2.65
Porosity ϕ	——	0.15	0.17
Permeability κ	darcy	0.5	0.8
Fluid density ρ_f	10^3 kg/m^3	1.0	1.0
Fluid viscosity η	10^{-3} Pa $\times$ c	1.0	1.0
Fluid bulk modulus K_f	10^9 Pa	2.25	2.25

each other on the frequency axis, while keeping f_{flow} at seismic frequencies.

By so doing, we have effectively cleared up the peaks of the fluid-flow attenuation from the influence of scattering and enabled the comparison of the numerical results for a poroelastic stack of layers with the theoretical predictions for this mechanism. We see that both for random (Figure 1a) and periodic (Figure 1b) sequences of poroelastic layers, the agreement between the numerical and theoretical results is satisfactory. An important feature of these results is that even for a random stack of layers, the numerically computed attenuation coefficients do not have random oscillations, up to a certain frequency, where for a random stack of layers the effect of scattering begins to build up. For a periodic stack, no complication due to the scattering is observed (as expected).

In Figure 2 we show the numerical results for a more realistic choice of parameters (permeabilities as given in Table 1; $h = 5$ m). In these examples, the theoretical peaks for the fluid-flow and scattering attenuation overlap considerably with each other, emphasizing the importance of the superposition model. In Figure 2a, we compare the numerical results for a random stack of layers with the superposition model. The overall agreement is quite good. However, the numerical results show random oscillations around the theoretical curve. Those oscillations represent the inherent feature of the scattering attenuation (Shapiro et al., 1994). The scattering nature of these oscillations is further evident from the comparison of Figure 2a with Figure 2b, where analogous calculations are shown for a purely elastic layered sequence (in the elastic case the sole mechanism of attenuation is scattering). The oscillations in the poroelastic case are very similar in character to those in the elastic situation, though their magnitude is lower in the poroelastic case.

The corresponding results for a periodic stack of layers are shown in Figure 2c. As has been predicted, scattering has no effect in the periodic case, up to the first band-stop frequency. The sharp peaks correspond to stopping bands as discussed in the theoretical section above.

As can be observed in both the random and periodic cases, at very high frequencies the standard Biot attenuation begins to build up. The agreement of the numerical results with the superposition model remains good.

CONCLUSIONS

We have studied the effect of fine layering on the attenuation of compressional waves propagating through a finely layered poroelastic medium (normal to layering). The theoretical model proposed assumes that the attenuation is caused by the three mechanisms: (1) standard Biot's attenuation, (2) scattering by fine layering, and (3) fluid flow between layers induced by the passing wave.

For random layering, the theoretical and numerical results show that (1) in the seismic and sonic frequency range the scattering and interlayer flow attenuation mechanisms dominate over the classical Biot attenuation; (2) the attenuation peak for the interlayer flow attenuation occurs at lower frequencies than that for the scattering attenuation; (3) the frequency dependence of the interlayer flow attenuation is more gradual than that for scattering or other known mechanisms of seismic attenuation; (4) unlike the scattering attenuation, the interlayer flow attenuation, though essentially depending on the statistics of random layering, does not have any oscillations around the (average) theoretical solution; (5) the *average* frequency dependence of the attenuation caused by the combined effect of the three mechanisms may be expressed as a superposition of the theoretical solutions for each of the mechanisms; (6) the random *oscillations* of the scattering attenuation around the average curve are lower in the poroelastic than in the corresponding elastic case. In other words, the presence of the interlayer flow attenuation decreases the level of the oscillations of the scattering attenuation.

For periodic layering, the theoretical solutions obtained on the statistical basis match both the known exact solutions and the results of numerical modeling.

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